CHAPTER VI- TN 14: DISTANCE AND THE 'REACTION' TO DISTANCE AS A FUNCTION OF DISTANCE

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ABSTRACT

"Distance and the 'reaction' to distance as a function of distance" is a phrase designed to emphasize the need for more analysis of the behavioural significance of gravity functions. This paper, in pursuing the behavioural significance theme, concentrates on research related to the "inertia models" suggested by Wolfe. The most important concept presented is that if an inertia model of travel behaviour of the type Wolfe (1972) describes is accepted, then the decision to visit a given location must involve a reaction to distance in marginal rather than absolute terms.

To quantify "reaction", the paper focuses on the effect of going one more unit of distance beyond a given point. This quantification being carried out leads to the conclusion that analysis of reaction to distance in marginal terms should be based on the properties of an "impedance due to distance function", IDF(d). A discussion of five gravity functions provides a basis for illustrating points and methods introduced. IDF(d)'s are derived for the functions considered. Conjectures on the response to distance suggested by the functions are checked by reference to the impedance of distance functions derived for the gravity functions introduced.

Conclusions relate to the fact that it is easy to misinterpret the significance of gravity functions as to how they imply decisions are made if one only looks at the shape of such functions. It is shown that in cases where one may think each new mile to be travelled offers more resistance than the last, each new mile offers less resistance or constant resistance.

PURPOSE

This paper presents a theoretical discussion and findings on "distance travelled as a function of distance". The purpose is not to fit functions to data but to clarify the behavioural interpretations that should be applied to functions which appear to fit available data.

INTRODUCTION AND THEORY

Wolfe (1972) has discussed what is usually considered to be the "distance part of a gravity function". General discussion on gravity and potential models is adequately covered in such sources as Olsson (1965) and Wilson (1970). Ginsberg has formulated a Markov Renewal Model that embodies the inertia model concept, but only mathematically oriented readers would find Ginsberg's formulation (1973) easy to follow. A paper in the recreation area employing a Markov Renewal Model is Gorman, Peterson and Lime (1972). Of course, Wolfe is not the only one (or even the first) to conceptualize the effect of distance on length of trips, although his explicit discussion of distance travelled as a function of distance is a seminal contribution. In fact, inertia effects are implicit in a variety of formulations. Stouffer's formulation of "intervening" opportunities (1940) introduces "inertia" effects resulting from the alternative opportunities encountered. Research on geographic and social mobility has resulted in discussion of the "principal of cumulative inertia" as explaining decreasing mobility rates with increasing length of stay in a community, profession or some other state (McFarland 1968; McGinnis 1970).

Discussions of distance travelled as a function of distance implicitly consider a function that is related to the probability of moving from being a traveller to being a visitor at a distance d from an origin. If one accepts the notion that such a function exists, the claim can be made that an impedance of distance function is of fundamental structural importance in understanding travel behaviour. For example, picture inertia in Wolfe's terms by viewing the decision making process as involving the thought: "If I have gone as far as d, what is my desire to go further?" And given the question, consider the expression:

$P(d,\Delta)$ = Probability of not going Δ units further than d

$$= \frac{\text{\# going at least to d but not to d+}\Delta}{\text{\# going to d or further than d}}$$

Since the area under the gravity function between d and d + Δ "measures" the number of persons who stop in that interval, while the area from d to infinity measures the number that go beyond d:

$P(d,\Delta) \approx \Delta(g(d))/\int g(x)dx$

WHERE the integration is from x=d to infinity (∞);

Δ is a relatively short distance (e.g. mile or Less for visits to major non-urban parks); and g(d) is the distance part of a gravity function that is defined so that the integral exists and defines the number of parties, vehicle or whatever units travelling to about d (e.g., observations would be $g(d) \approx$ number travelling to within $d\pm0.5$.

By allowing Δ to approach zero, one sees that an instantaneous measure of the impedance of distance is defined by:

(3) $IDF(d) = g(d)/\int g(x)dx$ WHERE the integration is from x=d to ∞.

From the way IDF(d) is defined, it is clearly reasonable to argue that IDF(d) is related to the resitance felt by a traveller toward going an additional time-distance unit, i.e. it is related to making a decision in marginal terms. It *is* not unreasonable to propose that IDF(d) is directly related to the marginal utility of continuing on from *d*. So, a decision in absolute terms is a decision in terms of the d's to certain destinations. For example, assume a person sees two physically identical sites at d_l and d_2 *as available.* In absolute terms one visualizes an "economically rational" person as choosing the closer site, or possibly suspects the two sites get visitors from the given origin in the ratio $g(d_1)/g(d_2)$.

However, given the concepts of "marginal" and "absolute" decision making, it is reasonable to argue that travel decisions involve both a marginal and an absolute component. Single purpose visits, particularly involving routine household functions (e.g. getting groceries) may involve a great deal of absolute "economic rationality" and little of the element, "if I go as far *as* X, I might as well go to Y". On the other hand sightseeing or vacation travel may be heavily weighted toward decisions based on marginal utility considerations. Wolfe's inertia concept applies primarily to this latter kind of travel. It is fair to say that this paper investigates a "polar" type of travel decisions, namely decisions made in marginal terms.

THE IMPEDANCE OF DISTANCE FUNCTIONS FOR SEVERAL g(d) FUNCTIONS

Although it is readily acknowledged that little is known on how decisions are made by various people for various types of trips, the model just proposed is used to see how a homogeneous group of people might react to distance if (their travel decisions were made in marginal terms and if their g(d) were known. And, since there exists a variety of g(d) functions that could be the correct g(d) function for *a* group, there are g(d) functions available that may reasonably be considered.

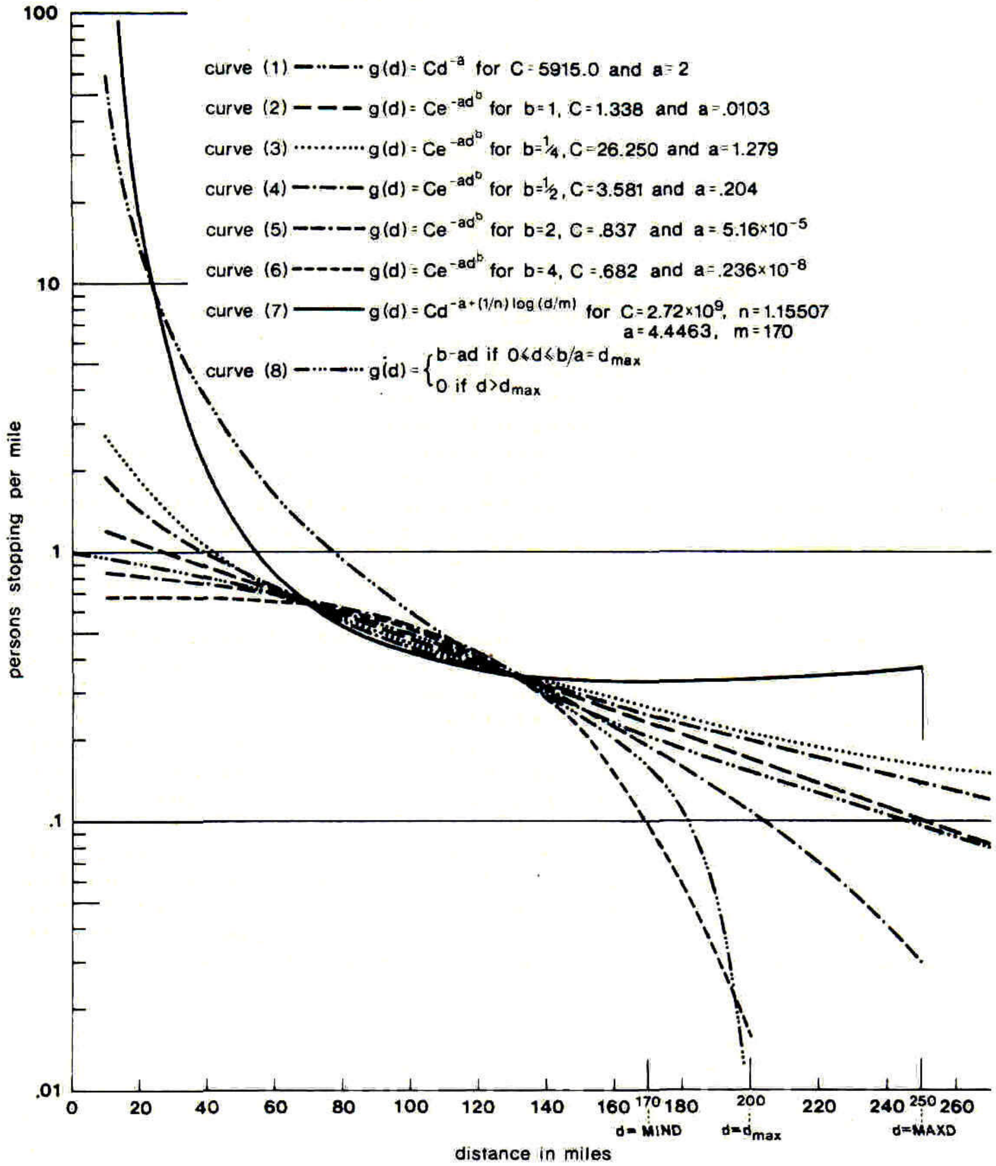
Five functions that have been considered in modelling travel behaviour are defined by Equations 4 through 8. Some of these functions are shown in Figure 1. The "standard" gravity model is based on:

(4) $g(d) = Cd^{-a}$

Unpublished CORDS research suggested:

(5) $g(d) = Cd^{-ad}$

Figure 1: Distance Component of Gravity Functions*



* The parameters of the functions were intentionally selected so that the functions would intersect at 70 an d130 miles, except for (1) which for a=2 can only be forced through one point.

A form used by the U.S. Corps of Engineers' equations for reservoirs is defined by:

(6) $g(d) = C \exp(-ad^b)$

Beaman and Leicester (see Reference 1) presented a variety of examples using linear functions to give potentials to visit. The following expression, while not the general case, is representative of the linear class of functions.

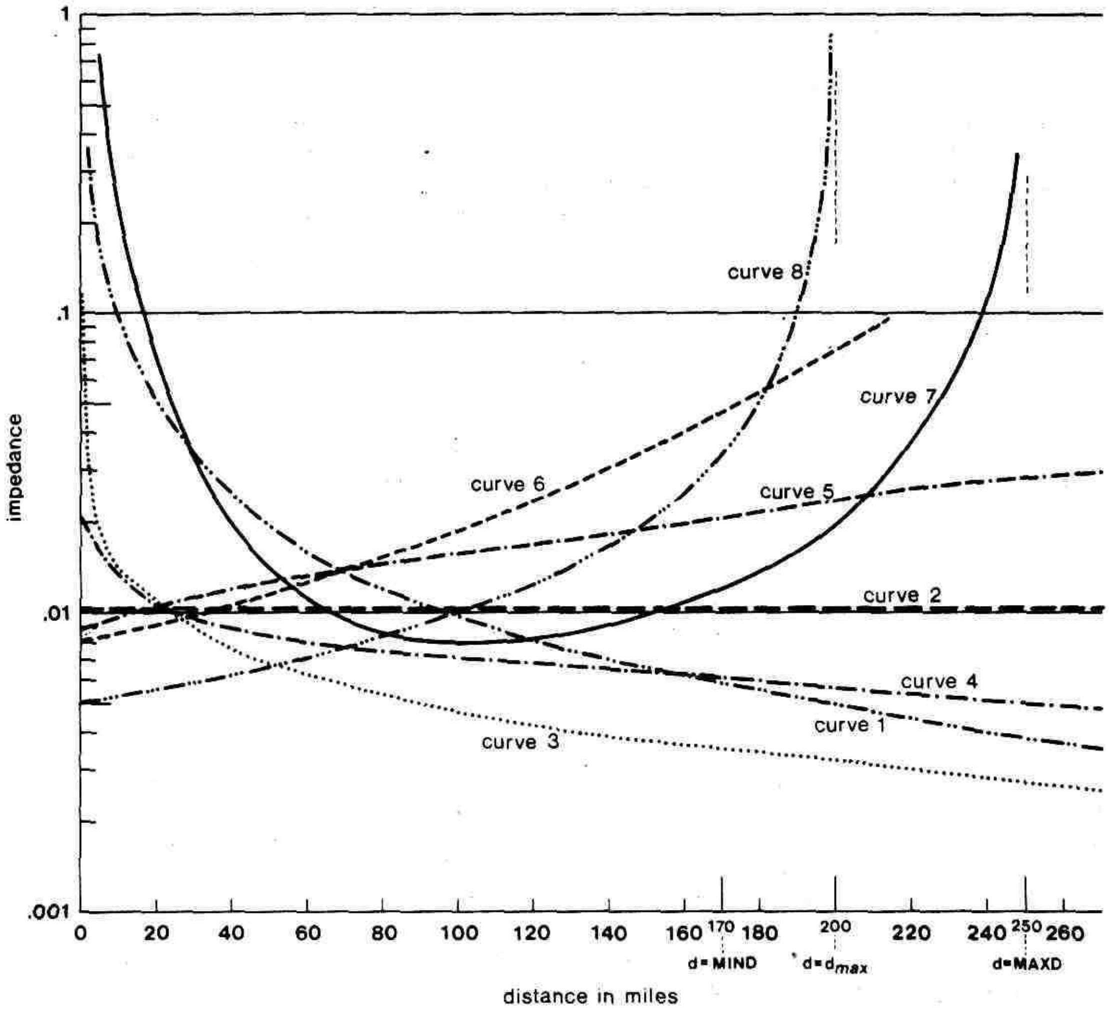
(7) $g(d) = b - ad$ with $b>0$ and $a>0$ up to a maximum distance, $d_{max}=b/a$ and 0 if $d>d_{max}$

Finally, the Wolfe "inertia" function (1972) is:

(8) $g(d) = Cd^{(-a+(1/n)\log(d/m))}$

WHERE *a* is the "usual" gravitational distance exponent and $n>0$ and $m>0$ are constants and it may be considered that $(1/n)\log(d/m)$ only applies for $d>m$.

Figure 2: Impedance of Distance Functions for the Functions Given in Figure 1



CONJECTURES

The five forms of g(d) introduced above embody a wide variety of implications about the effect of distance on the decision to travel beyond a point which has been reached. So before explicitly examining the IDF(d) functions for the equations introduced, some conjectures on the nature of the "impedance functions" expressing the "marginal utility" of additional travel are valuable because they illustrate how confused a researcher's impressions of what a person's reaction to distance may be when based on knowledge of g(d) rather than IDF(d).

Conjecture 1. The linear form (Equation 7) is associated with a constant force (impedance) acting in opposition to further travel.

Conjecture 2. An exponential form of g(d) implies an impedance, a force to stop, that increases with distance. (See Equation 5.)

Conjecture 3. The Cd^{-a} function implies that the further one has gone, the less likely one is to go another mile.

INTERPRETATION OF THE IDF(d)'s

The relation presented in Equation 3 may be used to determine the actual form of IDF(d) for the five g(d)'s introduced above. It is recognized that the way that IDF(d) is defined means it is similar to the force of mortality function considered in demography but this fact is not considered or exploited in the paper. So, using Equation 3 the IDF(d)'s were derived and are shown in Figure 2. The "traditional" gravity function has an IDF(d):

(9) $IDF(d) = (Cd^{-a})/(Cd^{1-a}) = (a-1)/d$ where $a>1$ so

In fact, ad hoc regressions that suggest that a is less than or equal to one implicitly suggest that an infinite number of visitors go further than any distance one may state. This is a "anamolous" condition researchers should consider as showing the distance function chosen is incorrect. In the second case:

(10) $IDF(d) = (Ce^{-ad})/(Ce^{-ad}/a) = a$

Equation 10 is Equation 11 with b equal to 1. The IDF(d) of the generalization of Equation 10 is different than the preceding since the integral in its denominator cannot be written in terms of a "simple" familiar function.

(11) $IDF(d) = C\exp(-ad^{b})/\int C\exp(-ad^{b})dx)$

WHERE the integration is from x=d to infinity; and if b = 2 the integral can be evaluated using tables for the normal distribution and where, if b is greater than zero, tables for the incomplete Gamma function allow evaluation of the function.

The IDF(d)'s were calculated using tables prepared for use in statistical work with the Pearson Type 3 distribution (see Reference 21). Specifically the integral $\int\exp(ax^{b})dx)$ may be transformed to a usual Gamma function form using $Y = ax^{b}$. In retrospect, it seems clear that IDF(d)'s could have been determined most easily if available Gamma Function values (integrals 0 to infinity) had been used with numerical routines to decrease them appropriately to get the values of the integrals of g(d) for increasingly larger values of d.

The IDF(d) for the "linear" g(x), takes a simple form:

(12) $IDF(d) = (b-ad)/(.5a(d(max)-d)^{2})$

where the denominator is recognized as the area of the triangle defined by a vertical line at some distance d on the x-axis, the x-axis and the line defined by Equation 7.

Strangely enough Equation 8 is of such a form that for m greater than one and n greater than zero (as Wolfe suggests they should be, so he gets his inertia effect) the area under the curve from d to infinity is "infinite". So, to have a usable function, a maximum distance to be travelled must be stated (e.g. 250 miles as in Figure 1) and then it is reasonable to write the following:

(13) $IDF(d) = Cd^{(-a+(1/n)\log(d/m))}/\int g(x)dx$

WHERE the integration is from x=d to 250 miles and the function must be evaluated by numerical methods for different values of a, n, and m.

The reader should note here that a variety of problems related to the existence of certain integrals have been ignored because it is not the object of this paper to present a comprehensive treatment of mathematical problems. However, it is possible to use Lebesque-Stieltjes integration to associate a measure with the point d equals zero so it becomes plausible to write of (as Wolfe does) start-up inertia. Start up inertia involves "mass" associated with a point not an interval, just as finite probabilities are is associated with integers for may distributions (e.g. of number of times, tosses)

One particular point should be noted regarding Wolfe's function because Equation 8 may be written as below:

(14) $g(d) = Cd^{-a-(1/n)\log(m)}\, d^{(1/n)\log(d)} = g(d) = Cd^{-A} d^{(1/n)\log(d)}$ where $A=a-(1/n)\log(m)$

When viewed this way, one sees that the inertia component of Wolfe's function is expressed by the (1/n)log(d) part of the exponent of the distance part of his gravity function. A number of other points may also be noted:

1. Wolfe's function is not the only function considered for which impedance decreases as distance increases.

2. The (1/n)log(d) component of the Wolfe function does initially serve to introduce a larger decrease in impedance than for a corresponding d^{-a} function.
3. At some arbitrary point, impedance begins to increase and then subsequently becomes infinite in a discontinuous and arbitrary way (e.g. as opposed to the linear form in which impedance becomes infinite but in a continuous and "understandable" way).
4. The Wolfe inertia function might as well be written as $Cd^{-A+b(\log(d))}$ since the m is only a parameter that should be "absorbed" into the gravity function because not both m and a can be estimated in a regression (since when A and b are determined, a and m are not uniquely determined by Equation 15).

Given the above points, it is fair to suggest that Wolfe's function involves an arbitrary and ad hoc correction to the usual distance part of a gravity function : as Wolfe suggests, it is a first correction.

As may have been suspected, conjectures on the kind of impedance function that would be associated with a given gravity function were constructed so that all conjectures present an incorrect view. The linear "gravity function" involves a situation where the force to stop increases with distance. In fact, as one approaches the point dmax equals b/a beyond which people do not go, the force function approaches infinity (see Curve 8, Figures 1 and 2). The exponential function considered does not suggest that each new mile one faces is viewed as offering a greater (or lesser) impedance than the last mile. Because of g(d) being the exponential "gravity function", IDF(d) equals a constant; it implies that every new mile is "like" the last one (see Curve 2 of Figures 1 and 2). Regarding the third conjecture, the distance part of the traditional gravity function C/d^a does not imply that each new mile presents a greater impedance than the last. The IDF(d) function, as shown by Curve 1 in Figure 2, is a function decreasing from very large values near zero to approach zero as d becomes large: Each new mile looks easier than the last and at large distances, each new mile presents almost no resistance to further travel compared to the early miles travelled.

As noted earlier, the equation used by the Corps of Engineers is a generalization of the simple exponential form in that $C^{-ad} = C\exp(-ad^b)$ if b = 1. It is particularly interesting because its IDF(d) function can have three types of shapes:

1. For b less than one it has the form shown by Curves 3 and 4 of Figure 1.
2. For b equals one it has the form given by Curve 2 of Figure 1.
3. For b greater than 1 it has the form given by 5 and 6 of Figure 1.

Thus, when the IDF(d) function for the Corps of Engineers' function (see Equations 11 and 14) is considered, three forms of the curve must be recognized. Firstly, if b is less than one, it is seen from Figure 2 that IDF(d)'s similar in shape to the one for d^{-a} are obtained. If the b in the Corps' equation is one, the equation becomes the simple exponential C^{-ad} for which the impedance function is a constant, as already noted. Finally, if b is greater than one, the IDF(d) function is similar in shape to the impedance function of the linear form of the distance part of a gravity function. As mentioned above, the impedance function for the linear form goes to infinity as distance approaches a maximum distance, d_{max}, that people travel according to the "linear gravity function". In this regard, the Corps' IDF(d) function, for b greater than one, does not go to infinity, for a finite d, but Figure 2 does show that for b equals 4, IDF(d) "takes off for infinity" very rapidly.

The IDF(d) for Wolfe's "inertia" gravity function is unique among the functions considered here because of its shape. When MAXD is defined as 250, as it is for the Wolfe function shown in Figure 1, the IDF(d) function, Curve 7 shown in Figure 2, drops initially showing a decreasing perceived impedance of distance as distances increases: in Wolfe's terms, an inertia to continue to travel is built up. But, as suggested by the IDF(d) function, ultimately the impedance begins to increase and then rapidly goes to infinity (e.g. in the example at MAXD = 250 miles).

In Wolfe's "inertia" gravity function, (if something is not plausible,) it is the form of the g(d) function. If the parameters of Wolfe's function are such that a minimum of g(d) occurs (in Figure 1 the minimum is at MIND = 170 miles), there is a point, the minimum of g(d), beyond which increasing numbers of people stop at each mile up to an abrupt boundary beyond which there are no trips. But, unless there is really some physical boundary to be considered in a particular problem, it is reasonable to suggest that no such abrupt discontinuity in g(d) should occur. In this regard, Wolfe's function might better be defined by:

$$g(d)=Cd^{-a+b(exp((-.5(d-u)**2))}$$

SUMMARY

From the curves presented in Figure 2, one notes that there are IDF(d)'s that (1) increase, (2) decrease, (3) are constant, and (4) both increase and decrease. The variety of shapes of IDF(d)'s shown in Figure 2 was certainly not expected by the author. Possibly, the most unexpected results found, however, relate to the fact that not all IDF(d)'s become infinite, showing that at least beyond (some distance) further travel becomes less desirable in the sense that the next mile offers more resistance than the last -each has higher impedance than the last. In fact, as already cited twice, as well as having an IDF(d) that is constant, functions commonly used in modelling were seen to have impedance functions that approached zero as distance travelled increased. One may note that decreasing impedance suggests that the fact that g(d) and the integral of g(d) from d to infinity both approach zero does not imply the increasing disutility of travel. Rather, the fact that some IDF(d)'s approach zero as d approaches infinity may be taken to reflect the gradual filtering off of people from a finite universe, even though the impedance to further travel continues to decrease.

CONCLUSION

In eliciting an intuitive feeling for his inertia model, Wolfe uses such phrases as: 'In the present note a suggestion is offered for one of these deficiencies - the unresponsiveness of the gravity model to the effect that distance itself has upon the perception of distance. When trips are very short, the friction of distance is negligible. The number of short trips, however, may be smaller than expected because a great many people may not wish to make a trip of any length, however short; their starting-up inertia, as it were, is too great to overcome. At the opposite extreme, one might hypothesize that, among the minority of people who indulge in lengthy trips, a still smaller minority find travel itself so stimulating that the further they go, the further they want to go.'

These statements imply that certain travel decisions are made in marginal terms; in terms of one's reaction to each additional time-distance unit to be travelled (in a manner implied by the IDF(d) function).

The point in recognizing marginal or some other decision making is recognizing that the most prominent lack in discussions employing gravity models is an inadequate behavioural basis for the gravity function introduced. However, this is not to say literature in which behavioural considerations play a role is nonexistent. But, for example, one should not consider Wilson's "Interpretation of Terms Section" of his book (1970) as the point at which behaviour is introduced into the entropy-transport model. Human behaviour in so far as it is relevant was introduced as implicit in the entropy framework. It is interesting to note Simon's discussion of learning theory models (Simon 1957) where he shows that a theoretically derived equation' which is the case for Wilson's gravity model equation, may be consistent with a number of drastically different behavioural theories. One may also see Cesario (1973) where he discusses why one of Wilson's more unrealistic assumptions may be relaxed to be more realistic without affecting the model one would use to explain travel behaviour.

From a theoretical perspective the significance of this note is clearly considered to lie in its focus on the behavioural significance of the IDF(d) function. The author frankly was surprised by the variety of IDF(d) functions resulting from the consideration of g(d)'s which are so similar over the range of d's often considered. (See Note at end of text.) However, when the significance of IDF(d) is recognized, research may be designed to allow more adequate definition of IDF(d) and thus, g(d).

From a broader perspective it is hoped that this discussion of marginal decision making prompts research on the "mix" of marginal and absolute elements in various travel decisions. Discussion and research are needed because, although formulas resulting in good aggregate predictions of trip distribution are valuable for certain applications, manipulation of these formulas without asking why they work is not social science research. Social science research must ultimately relate accepted formulas that predict behaviour to fundamental social, psychological and, possibly, biological processes.

Note: The similarity of different g(d)'s implies that the choice of a given g(d) on the basis of R^2 or another criterion would involve a high probability of an incorrect choice (of Type 2 error). This should be recognized in evaluating the significance of results by Wilkinson (1973). Actually, any of the gravity functions considered here deviate from d^{-a} in such a way that in Wilkinson's terms an inertia effect may be found. Also, one should recognize that Wilkinson is dealing with Wolfe's function, which suffers from the other problem already noted.

The author believes that Wilkinson was estimating parameters that could not be uniquely estimated without making unwarranted assumptions. In fact, Wilkinson's introduction of m = 1/8 into his model was not based on objective observation of visitor behaviour. Only when m is determined experimentally should it be regarded as anything but an arbitrary and ad hoc correction to a. On this matter, one may also see that points made about Equation 15 imply that there are an infinite number of parameter sets, one for each m (e.g., m equals 1/8, 1/4, 1/2, 1.0, 2.0, etc.) that give the same explained sum of squares in the kind of regression that Wilkinson carried out - thus his parameterized model is not really based on scientific observation.